J. Fluid Mech. (1977), *vol.* 81, *part* 3, *pp.* 421–431
Printed in Great Britain

An experimental investigation of flow in an oscillating pipe

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(Received 10 December 1975 and in revised form 4 November 1976)

The hydrogen-bubble technique has been used to measure the velocities of pulsating water flow in a rigid circular pipe. Mean flows with Reynolds numbers between 1275 and 2900 were superimposed on an oscillating flow produced by moving the pipe axially with simple harmonic motion. While the velocities in the oscillating boundary layers on the pipe wall were found to be close to those predicted by laminar flow theory, at the higher Reynolds numbers the velocities near the centre of the pipe were lower than those predicted and more uniformly distributed.

The intermittency of the periodic bursts of turbulent motion at the higher Reynolds numbers was measured. At each mean-flow Reynolds number the turbulent intermittency of the flow was found to be a function of a single parameter: the harmonic-flow Reynolds number.

1. Introduction

Sinusoidal pulsating flow in a rigid circular pipe may be described by the mean-flow velocity U_m, the frequency of pulsation ω and the magnitude of the harmonic velocity U_h. The corresponding dimensionless parameters are the mean-flow Reynolds number $Re_m = 2U_m R/\nu$, where R is the pipe radius and ν the kinematic viscosity of the fluid, the frequency parameter $\alpha = R(\omega/\nu)^{\frac{1}{2}}$, expressed in (radian)$^{\frac{1}{2}}$, and the velocity ratio $\gamma = U_h/U_m$.

General mathematical solutions for laminar pulsating flows have been established by Womersley (1955) and Uchida (1956), but experimental verification has been attempted over only a limited range of the flow parameters, by Denison, Stevenson & Fox (1971). Studies of the transition to turbulence in pulsating flows in rigid pipes by Gilbrech & Combs (1963), Sarpkaya (1966) and Yellin (1966) measured the growth rate of turbulent slugs in the flow at frequencies up to $\alpha = 16$ and velocity ratios up to $\gamma = 1$, but have given no information on the velocity distributions in the flow. Velocity measurements have been made in physiological flows by Ling, Atabek & Carmody (1968) and Nerem, Seed & Wood (1972) but these have been limited to the flow conditions which can be achieved in the aorta, where the flow is not fully turbulent.

The measurements of water-flow velocity distributions in a rigid circular pipe described in this paper were made in both laminar and turbulent flow with a maximum mean-flow Reynolds number $Re_m = 2900$. The flow pulsations were produced by oscillating the pipe axially at frequencies up to $\alpha = 26{\cdot}7$ and velocity ratios up to $\gamma = 5{\cdot}4$. The turbulent intermittency of the flow was investigated up to a frequency $\alpha = 32$ and a velocity ratio $\gamma = 6{\cdot}5$.

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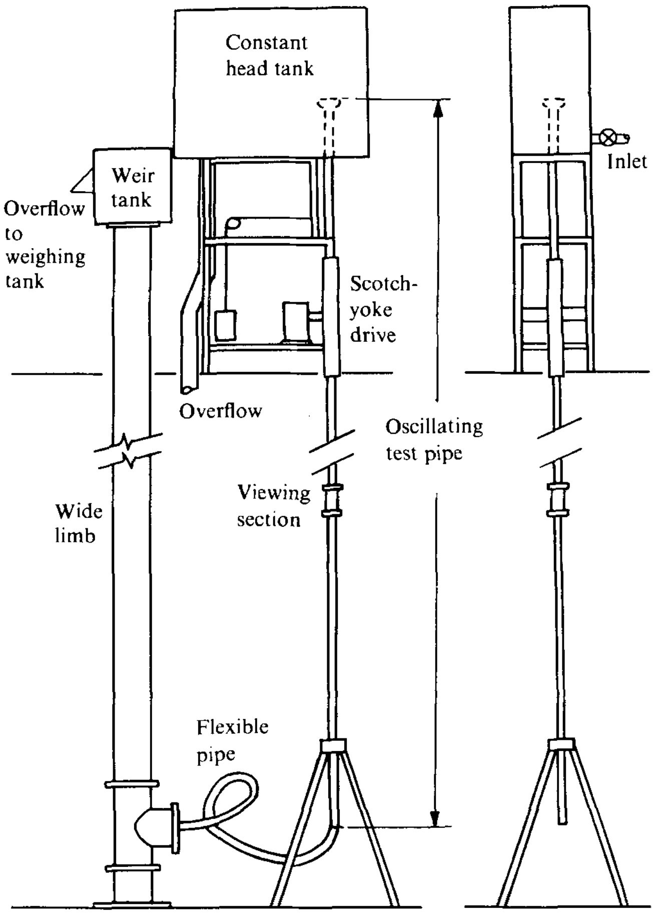

FIGURE 1. The oscillating-pipe apparatus.

2. The oscillating-pipe experiment

The flow circuit

A U-tube with limbs of unequal diameter had as its smaller limb a test pipe of internal diameter 51 mm and length 6 m (figure 1). The mean flow rate of water through the U-tube was controlled by the difference in surface levels between the constant-head tank mounted over the test pipe and the weir tank mounted over the wide limb. The test pipe was free to move axially in the constant-head tank and was oscillated with simple harmonic motion by a scotch-yoke mechanism. The motion of the test pipe, at frequencies up to 2 rad s^{-1} and amplitudes up to 148 mm, induced a flow which will be shown to be equivalent to that produced by a harmonic pressure gradient.

Motion in a pipe oscillating axially

Laminar motion in a long pipe can be represented by

$$\frac{\partial u'}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\left\{\frac{\partial^2 u'}{\partial r^2} + \frac{1}{r}\frac{\partial u'}{\partial r}\right\}, \tag{1}$$

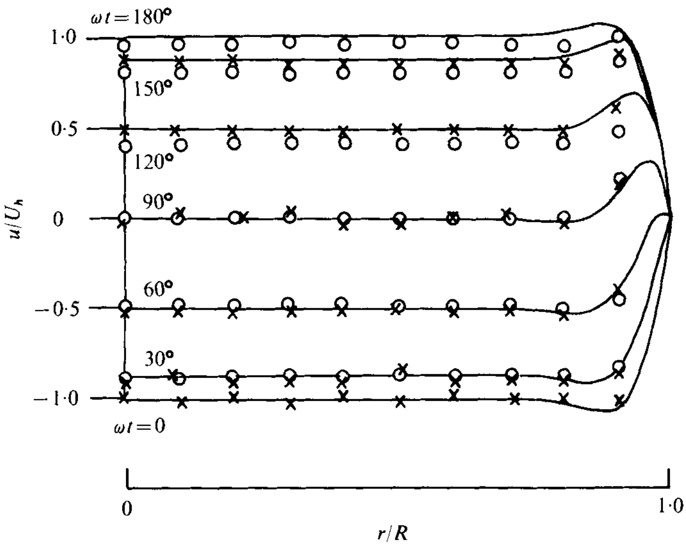

FIGURE 2. Theoretical and measured velocities in oscillating flow. Frequency $\omega = 1{\cdot}12\,\mathrm{rad\,s^{-1}}$, $\alpha = 26{\cdot}7$. Pipe amplitude X/R: ○, 5·8; ×, 1·45.

where x = axial co-ordinate, r = radial co-ordinate, u' = velocity in the x direction and ρ = fluid density. Considering the flow in the oscillating pipe and defining u as the velocity of the flow relative to the pipe yields

$$u = u' - U_h \cos \omega t, \tag{2}$$

where $U_h = X\omega$ and X = amplitude of pipe oscillation. After substitution of (2), (1) becomes

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial x}(1 + \epsilon \sin \omega t) + \nu\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r}\right), \tag{3}$$

where $\epsilon = -\rho\omega U_h(\partial p/\partial x)^{-1}$ is equivalent to the dimensionless amplitude of a harmonic pressure gradient. The boundary conditions for a circular pipe are

$$\partial u/\partial r = 0 \quad \text{at} \quad r = 0,$$
$$u = 0 \quad \text{at} \quad r = R,$$

and the solution is similar to that derived by Womersley (1955):

$$u/U_m = 2(1 - r'^2) - \gamma \cos \omega t + \gamma M \cos(\theta + \omega t), \tag{4}$$

where $$M = M_0(\alpha r')/M_0(\alpha), \quad \theta = \theta_0(\alpha r') - \theta_0(\alpha), \quad r' = r/R.$$

$M_0(z)$ and $\theta_0(z)$ are the modulus and phase of Bessel functions of zero order:

$$M_0(z) = (\mathrm{ber}_0^2(z) + \mathrm{bei}_0^2(z))^{\frac{1}{2}}, \quad \theta_0(z) = \tan^{-1}(\mathrm{bei}_0(z)/\mathrm{ber}_0(z)).$$

The three terms in (4) represent a Poiseuille flow, the motion of the pipe and the flow in the oscillating boundary layer (Stokes layer) respectively. The equation is plotted on figures 2–4 for comparison with points representing measured velocities. Figure 2 shows typical velocity profiles across the pipe for half a cycle of oscillatory flow. When the pipe is at rest ($\omega t = 90°$) the motion is solely that due to the thin Stokes

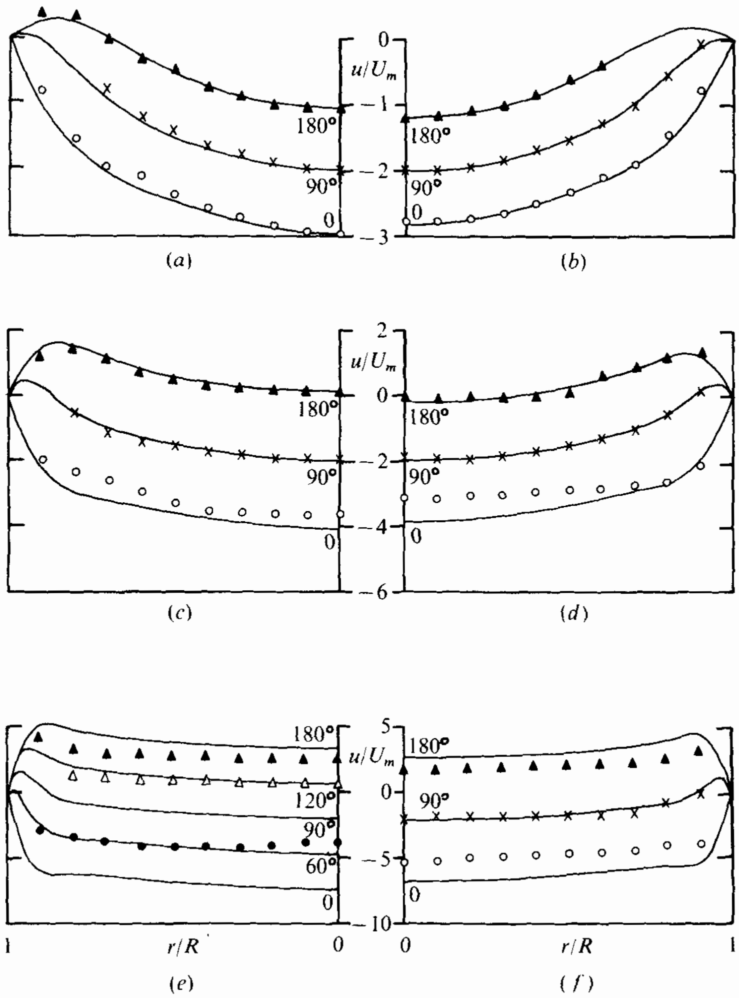

FIGURE 3. Theoretical and measured velocities in pulsating flow. Mean-flow Reynolds number: (a), (c), (e) $Re_m = 1535$; (b), (d), (f) $Re_m = 1785$. Frequency parameter: (a), (b) $\alpha = 11{\cdot}2$, $Re_h = 1460$; (c), (d) $\alpha = 16{\cdot}6$, $Re_h = 3210$; (e), (f) $\alpha = 26{\cdot}7$, $Re_h = 8300$. Values of ωt for experimental points: ○, 0; ●, 60°; ×, 90°; △, 120°; ▲, 180°.

layer near the wall of the pipe. In pulsating flow, figures 3 and 4, the mean-flow velocity in the test pipe is downwards, i.e. U_m is negative, and when the pipe is at rest ($\omega t = 90°$) the flow velocity outside the Stokes layer has a Poiseuille velocity profile with $u/U_m = -2$ at the centre-line.

Velocity measurement

In many of the experiments the oscillation of the pipe produced a value of γ which was sufficient to cause complete reversal of the flow relative to the pipe. Hence the method of velocity measurement had to indicate the flow direction with certainty and with this in mind the hydrogen-bubble technique described by Schraub *et al.* (1965) was used. This method of measurement was found to have the additional advantage of being sensitive to disturbances in the flow during transition.

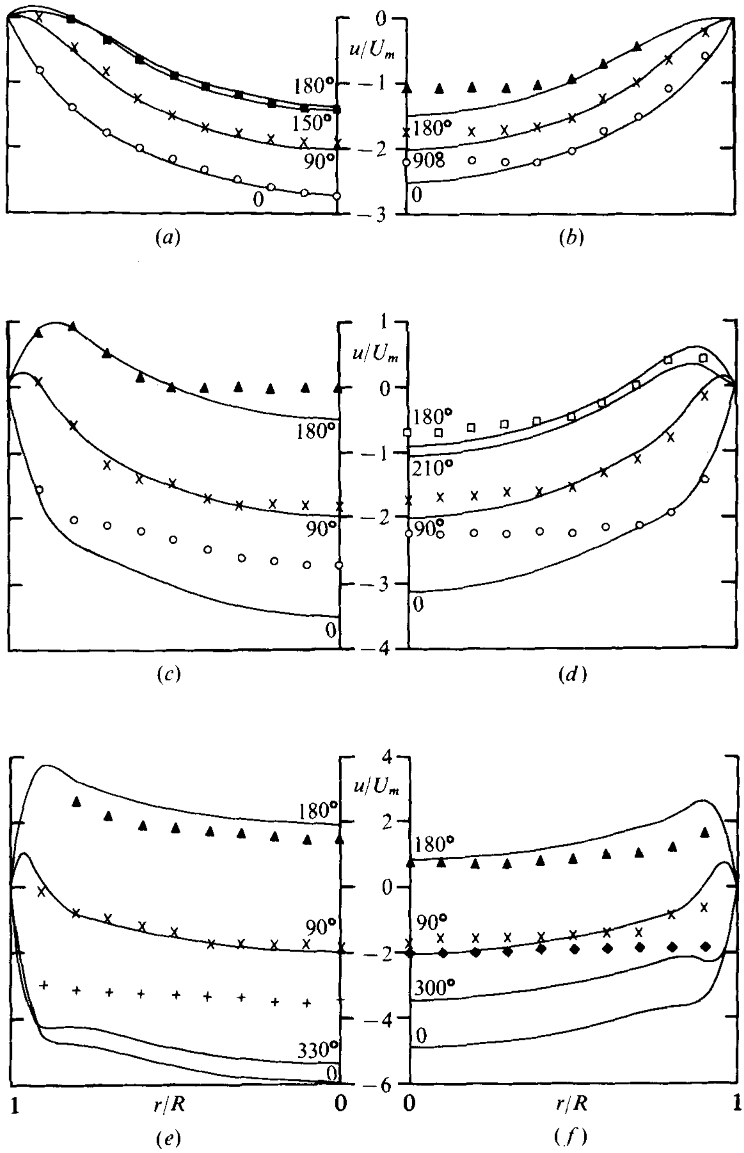

FIGURE 4. Theoretical and measured velocities in pulsating flow. Mean-flow Reynolds number: (*a*), (*c*), (*e*) $Re_m = 2115$; (*b*), (*d*), (*f*) $Re_m = 2900$. Frequency parameter: (*a*), (*b*) $\alpha = 11{\cdot}2$, $Re_h = 1460$; (*c*), (*d*) $\alpha = 16{\cdot}6$, $Re_h = 3210$; (*e*), (*f*) $\alpha = 26{\cdot}7$, $Re_h = 8300$. Values of ωt for experimental points: ○, 0; ×, 90°; ■, 150°; ▲, 180°; □, 210°; ◆, 300°; +, 330°.

A Perspex block machined internally to the same diameter as the test pipe was placed 60 diameters downstream of the inlet and three 0·08 mm diameter platinum wires were stretched across the diameter at 30 mm intervals along the length of the block. The wires were connected to an electrical pulse generator and the hydrogen bubbles produced in the flow were photographed by a camera attached to the pipe. The bubbles were illuminated by electronic flash equipment fired at a set point in the cycle by contacts on the drive mechanism.

The negatives showing the displacements of the bubbles relative to the pipe (figure 5, plate 1) were projected on a chart reader and the flow velocities calculated from the measured displacements of the bubbles and the generator pulse frequency. The points plotted on figures 2–4 show typical measured velocities for one half-cycle of the flow, the other half-cycle being omitted for clarity. All experimental results, except those on the pipe axis, are the mean of the two velocities measured at the same radius on opposite sides of the pipe axis, any profile with pronounced asymmetry being rejected for the measurement of velocities. Estimates of the errors due to the rate of rise of the bubbles and the wire wake by Clamen (1973) show that for laminar flow the velocity measurement will be within 6% of the actual velocity.

3. Experimental results and discussion

Oscillating laminar flow

Initial tests were performed on the apparatus by oscillating the pipe with no mean flow and the measurements of velocity compared with those predicted by Richardson & Tyler (1929). Up to a frequency of $\alpha = 26{\cdot}7$ and at the three amplitudes of oscillation used, $X/R = 1{\cdot}45$, $2{\cdot}9$ and $5{\cdot}8$, the measurements agreed with the predictions to within the 6% accuracy expected, e.g. figure 2. At higher frequencies and at the highest amplitude the discrepancy between the measurements and the theory increased to above 10% but this was apparently due to the difficulty of maintaining a true simple harmonic pipe motion when high mechanical power inputs were required to oscillate the pipe.

Even at the highest frequency, $\alpha = 34$, the hydrogen-bubble lines were symmetrical and indicated a laminar flow. This was expected as the maximum value of the stability parameter $A = 2X\alpha/R$ was 394 and thus below the critical value of 400 suggested by Merkli & Thomann (1975) for transition in oscillating pipe flow.

Velocity measurements in pulsating flow

Having established that the apparatus produced laminar oscillating flows up to a frequency $\alpha = 27$, the effect of adding a steady mean flow which could be laminar or turbulent was investigated. Five mean flows with Reynolds numbers between 1275 (laminar flow) and 2900 (turbulent flow) were superimposed on different oscillating flows with frequencies such that $11{\cdot}2 \leqslant \alpha \leqslant 26{\cdot}7$. To ensure that the oscillatory motion remained large relative to the mean flow rate, only the maximum amplitude of pipe oscillation $X/R = 5{\cdot}8$ was used, giving velocity ratios from $\gamma = 0{\cdot}5$ to $\gamma \doteq 5{\cdot}4$.

Visual observations and photographs of the hydrogen-bubble lines showed that flow instability was a periodic phenomenon repeated at the same time in each cycle and separated by periods of laminar flow. At any instant the pulsating flow could be undisturbed, disturbed or highly disturbed, in a similar manner to the flow in the aorta observed by Nerem *et al.* (1972). The undisturbed profiles, figures 5(*b*) and (*c*), were symmetrical, the motion of the bubbles being regular and the flow laminar. The disturbed profiles, figures 5(*a*) and (*f*), had regular bubble motion but were asymmetrical near the outer edge of the Stokes layer, while the highly disturbed profiles were so irregular that it became difficult to take mean velocity measurements from the photographs. The disturbed and highly disturbed velocity distributions would appear to correspond to the DL and DH disturbances observed by Clarion & Pélissier (1975) in oscillating flow.

The shape of the measured velocity profiles, of which figures 3 and 4 show typical examples, depended on the state of disturbance of the flow. In undisturbed flow, $\alpha \leqslant 11{\cdot}2$, $Re_m \leqslant 2115$, the velocities agreed closely with those predicted by laminar flow theory (figures 3*a*, *b* and 4*a*). Disturbed flow was first observed at values of $\alpha > 11{\cdot}2$ during those periods of the cycle when the velocity relative to the pipe was greatest, i.e. $330° < \omega t < 30°$. The disturbance resulted in a reduction in the flow velocity but little change in the profile shape, e.g. figures 3(*c*) and (*d*) at $\omega t = 0$. Increasing the frequency produced disturbed flow during the reversed-flow part of the cycle around $\omega t = 180°$ with similar reductions in velocity (figures 3*e*, *f*).

The flow became highly disturbed only above mean-flow Reynolds numbers of 2000. Under these conditions the velocities in the Stokes layer remained close to those predicted by laminar flow theory up to $\alpha = 16{\cdot}6$ but the velocities in the centre of the pipe were reduced and became more uniform, as in figures 4(*b*)–(*d*). The flattening of the velocity profiles, which has been observed to occur in oscillating flow at $\alpha = 13{\cdot}3$ by Clarion & Pélissier (1975), became even more pronounced at $\alpha = 26{\cdot}7$, the highest frequency used (figures 4*e*, *f*).

Transition to turbulence

Qualitative observations of the velocity profiles confirmed that the periodic nature of the disturbances recorded by Nerem *et al.* (1972), Yellin (1966) and others was also typical of the flow in the oscillating pipe during transition. The distortion of the hydrogen-bubble lines by the disturbances in the flow made it possible to measure the 'intermittency' of the flow λ, defined as the percentage of the time the flow was disturbed during a 5 min measuring period.

The intermittency of the steady flow with the pipe at rest was first measured and found to give a conventional transition curve; figure 6(*f*). Measurements of the intermittency of pulsating flow were then made at each of the previous five mean flow rates and at an additional flow rate corresponding to $Re_m = 2300$. Three amplitudes of pipe oscillation ($X/R = 5{\cdot}8$, $2{\cdot}9$, $1{\cdot}45$) were used at frequencies up to $\alpha = 34$. The variation of intermittency with frequency is plotted in figure 6, which shows that, for a given frequency of oscillation, an increase in the mean-flow Reynolds number or amplitude of oscillation produced an increase in the intermittency. However, if the mean-flow Reynolds number and amplitude of oscillation were kept constant, increasing the frequency of oscillation could, in certain circumstances, produce a reduction in the intermittency of the flow. With a laminar mean flow (figures 6*a*–*c*), the intermittency had a minimum at a value of α which depended on the amplitude but not on the mean-flow Reynolds number. With a transitional mean flow (figures 6*d*, *e*), the reduction of intermittency was smaller and it was not possible to identify a minimum value.

The stabilizing effect of the harmonic motion in pulsating flow has been recorded elsewhere. Gilbrech & Combs (1963) and Sarpkaya (1966) have measured increased values of the critical Reynolds number at frequencies up to $\alpha = 10$ and amplitudes up to $\gamma = 1$. For a frequency of $\alpha = 7$ and $\gamma < 0{\cdot}4$ Yellin (1966) measured a reduction in the intermittency similar to that in the oscillating pipe but at higher mean-flow

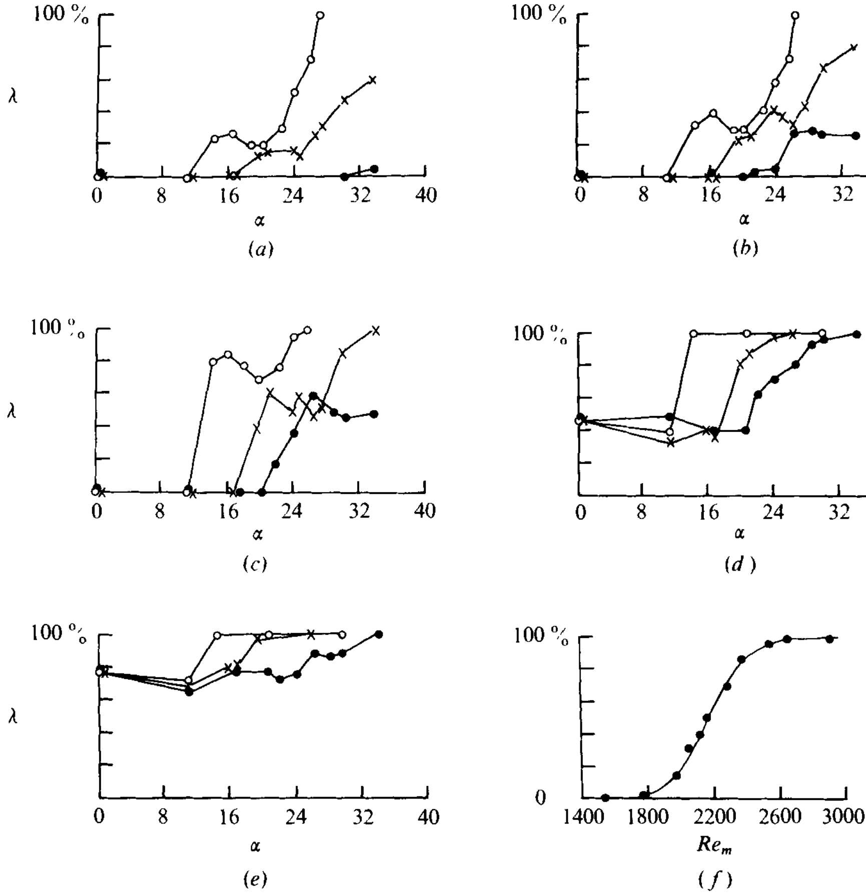

FIGURE 6. Transition to turbulence in pipe flow. Mean-flow Reynolds number for pulsating flow: (a) $Re_m = 1275$, (b) $Re_m = 1535$, (c) $Re_m = 1785$, (d) $Re_m = 2115$, (e) $Re_m = 2300$. Pipe amplitude X/R: ○, 5·8; ×, 2·9; ●, 1·45. At $Re_m = 2900$ flow is fully turbulent. (f) Steady flow transition.

Reynolds numbers. This difference may be due to the disturbed entry conditions caused by the motion of the oscillating pipe in the inlet tank.

Yellin demonstrated that the reduction of the disturbances in pulsatile flow depends on the time the flow spends below the critical Reynolds number. For a particular mean-flow Reynolds number this 'relaxation time' will depend on the Reynolds number Re_h of the harmonic flow, which in the present experiment may be defined as

$$Re_h = 2RX\omega/\nu = 2\alpha^2 X/R.$$

Plotting the intermittency against Re_h for each mean-flow Reynolds number (figure 7) showed some correlation of the observations taken at different amplitudes of oscillation. This enabled a three-dimensional surface to be drawn (figure 8) which approximately defined the intermittency λ as a function of only two parameters: the mean-flow and the harmonic-flow Reynolds numbers.

The apparent correlation between transition and the harmonic Reynolds number

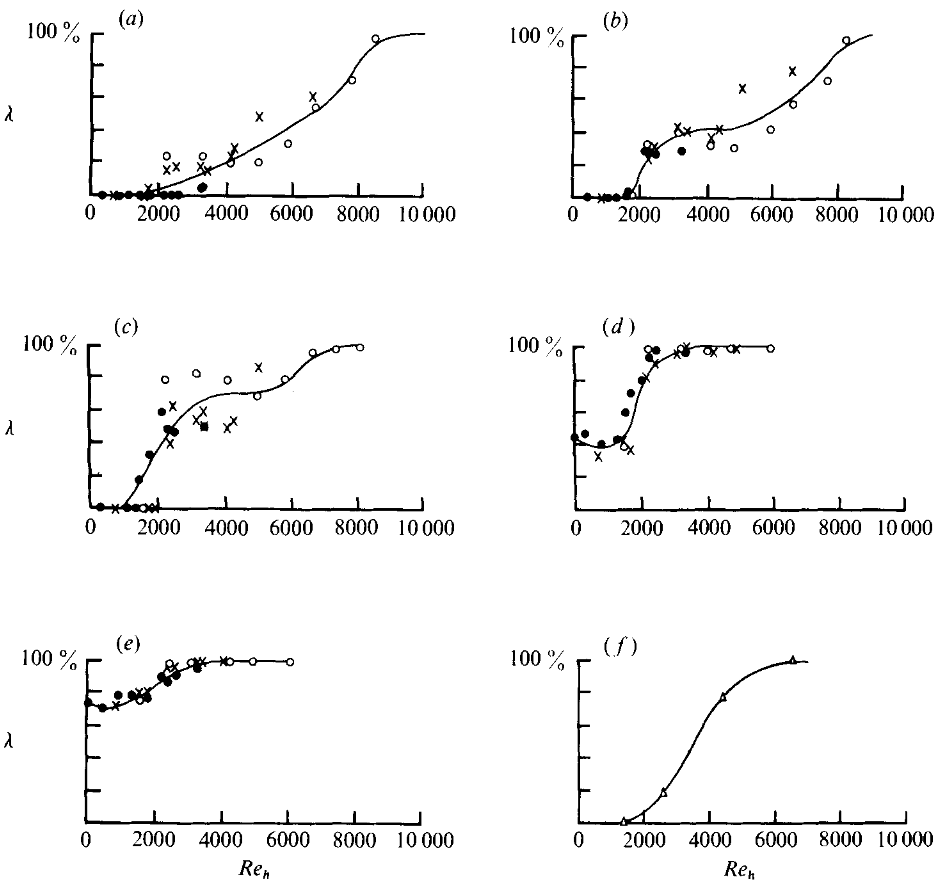

FIGURE 7. Transition to turbulence in pipe flow. Mean-flow Reynolds number: (a) $Re_m = 1275$, (b) $Re_m = 1535$, (c) $Re_m = 1785$, (d) $Re_m = 2115$, (e) $Re_m = 2300$, (f) $Re_m = 0$, after Clarion & Pélissier. Pipe amplitude X/R: ○, 5·8; ×, 2·9; ●, 1·45. At $Re_m = 2900$ flow is fully turbulent.

is at variance with the parameter A used by Merkli & Thomann (1975) to predict transition in oscillating flow, as

$$A = 2\alpha X/R = Re_h/\alpha.$$

With a laminar mean flow the values of A at which disturbances first appeared in the pulsating flow were 150 at $X/R = 5{\cdot}8$ and 60 at $X/R = 1{\cdot}45$, i.e. less than the critical value $A_c = 400$ for oscillating flow.

It is possible that the proposed correlation is appropriate only for the flow conditions in an oscillating pipe used as in the experiment. However, the suggested parameter may have wider applications as recordings presented by Clarion & Pélissier (1975, plates 2 and 3) can be used to calculate the intermittency of oscillating motion in a U-tube. When the intermittency is plotted against the harmonic Reynolds number (figure 7f), the curve obtained is similar in shape to the curves for pulsating flow in the oscillating pipe.

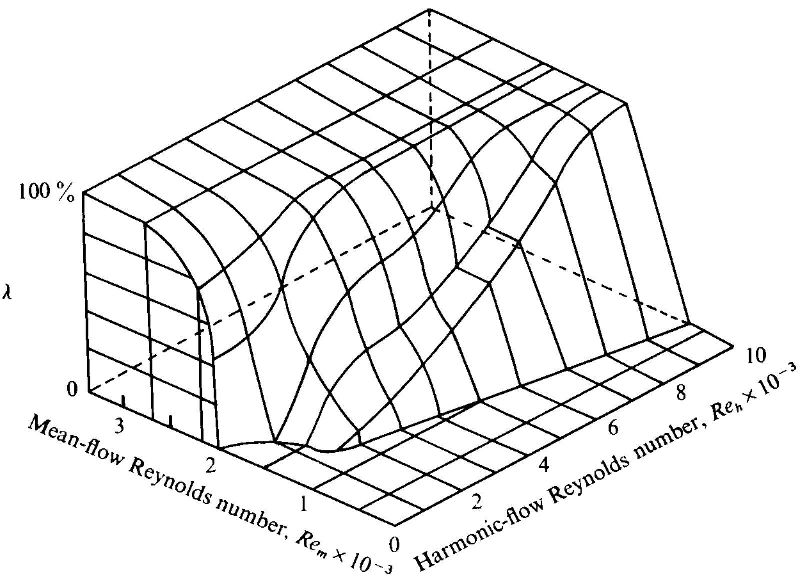

FIGURE 8. A tentative transition surface for pulsating flow in an oscillating pipe.

4. Conclusions

It has been demonstrated that the laminar flow in a rigid circular pipe oscillating along its axis is similar to that produced by a harmonic pressure gradient. With no mean flow the oscillatory motion in the pipe remained laminar up to the maximum value of the stability parameter reached: $A = 394$.

Pulsating-flow velocity distributions were found to correspond to laminar flow theory up to a mean-flow Reynolds number of 1500 and a frequency corresponding to $\alpha = 10$. At higher Reynolds numbers and frequencies there was an increasing difference between the velocities predicted by laminar flow theory and those measured in the central region of the pipe, the measured velocities being smaller and more uniformly distributed.

It was observed that in general the turbulent intermittency of the flow increased with the mean-flow Reynolds number and with the frequency and amplitude of the harmonic motion. However at mean-flow Reynolds numbers below 2000 there were frequencies where an increase in the frequency of the harmonic motion resulted in a small reduction of the turbulent intermittency of the flow. For the conditions of the experiment it was found that the turbulent intermittency of the flow could be defined by only two parameters: the mean-flow Reynolds number and the harmonic-flow Reynolds number $Re_h = 2\alpha^2 X/R$.

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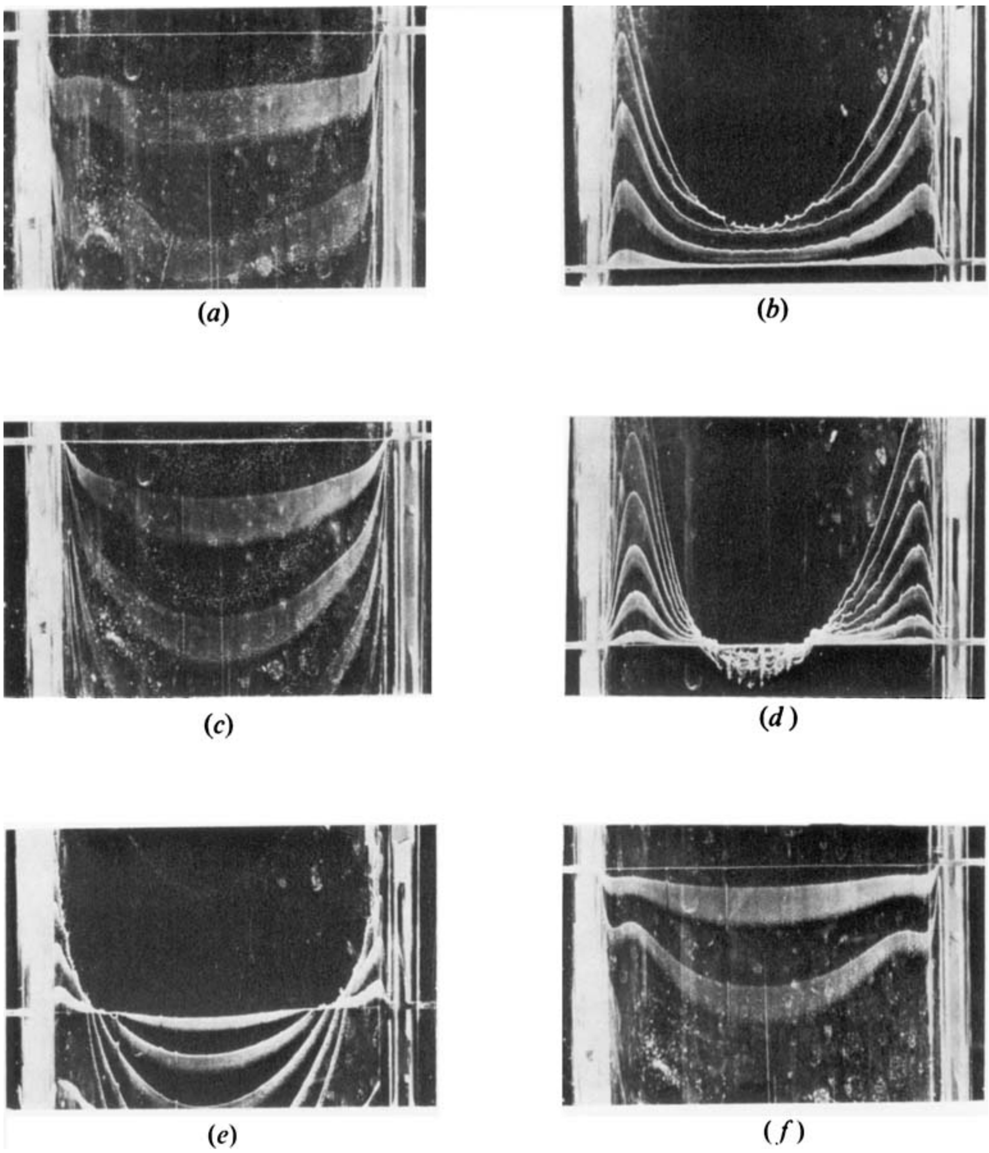

FIGURE 5. Hydrogen-bubble photographs in pulsating flow. (*a*) $\omega t = 0$, (*b*) $\omega t = 180°$, (*c*) $\omega t = 60°$, (*d*) $\omega t = 240°$, (*e*) $\omega t = 120°$, (*f*) $\omega t = 300°$. Mean flow down page. $Re_m = 1535$, pipe amplitude $X/R = 5{\cdot}8$, $\omega = 0{\cdot}549\,\mathrm{rad\,s^{-1}}$, $\alpha = 18{\cdot}7$, $Re_h = 4100$. Bubble line generation frequency $= 10\,\mathrm{Hz}$.